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Autorotation of rectangular plates

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A series of tests have been conducted on the autorotation of plates of rectangular section and thickness to chord ratios of from 0.1 to 1.0. The major difference from previous work was that the plates spanned the full width of the wind tunnel, i.e. the flow was essentially two-dimensional. The results show major differences from predictions of infinite aspect ratio plates inferred from finite aspect ratio tests as given by Iversen (1979). Moreover, they give excellent correlation with the two-dimensional numerical solution of Lugt (1980) for a 10% thick plate. In contrast to previous results which indicate no autorotation for square cylinders ($t/c = 1.0$), it is found that autorotation is easily achieved. A new result is that tipspeed ratios are found to be independent of thickness ratio, at approximately the Lugt value over the full range of thickness ratios. Drag coefficients are found to be independent of thickness ratio. There appears to be a critical thickness ratio above and below which the lift coefficients are constant, but are of different magnitude.

1. Introduction

Autorotation is the continuous rotation, without external power, of a body exposed to an airstream. Autorotational motion can take place either in free flight or when a plate is pinned to rotate about a given axis in a wind tunnel. A flat plate pinned to rotate about its mid-chord must be given an initial rotational impulse to autorotate.

The interest in autorotation and the range of studies conducted until recently is well covered in the review by Lugt (1983). The phenomenon was initially studied nearly 100 years ago by Maxwell who recognized that the centre of mass and centre of the aerodynamic forces do not coincide, thus giving rise to a torque. Riabouchinsky (1935) distinguished for the first time between autorotating plates with fixed axes and those with freely moving axes. More recently, Bustamante & Stone (1969), Iversen (1969), and Smith (1971) suggested that autorotation of plates of symmetrical cross-section is due to large vortices shed from the retreating faces of the cross-section.

All the main features, and many details of the flow, have been given by Lugt (1980), as well as clarification of the conditions which have to be met for autorotation to occur. He solved the Navier–Stokes equations numerically for a thin elliptical cylinder rotating at constant angular velocity in a two-dimensional airstream. Full solutions were obtained at a Reynolds number of 200, for a thickness ratio of 0.1. It was shown that autorotation occurred at a ratio of tipspeed to airspeed of 0.45. It was argued that a similar value should be obtained at higher Reynolds numbers. No solutions were found for an elliptical cylinder of 0.6 thickness ratio and it was concluded that the rounded edges weaken the shed vortices which drive the plate.

The experimental results of Bustamante & Stone (1969), Glaser & Northrup

(1971), and Smith (1971) have been correlated by Iversen (1979). He showed consistency of results between free-flight and wind-tunnel tests and was able to account for the effect of aspect ratio and thickness ratio on the tipspeed ratio. The effects of bearing friction were accounted for by taking the slope of the tipspeed versus airspeed curve, i.e. dV/dU, as the tipspeed ratio, rather than V/U. All the experimental results were for aspect ratios of between 0.25 and 4, and thickness ratios from 0.5 down. The correlation equation that he obtained for tipspeed ratio, based on this data is:

$$\frac{V}{U} = \left(0.329 \ln\left(\frac{c}{t}\right) - 0.0246\left(\ln\left(\frac{c}{t}\right)\right)^2\right)\left\{\left[\frac{A}{2+(4+A^2)^{\frac{1}{2}}}\right]\left[2-\left(\frac{A}{A+0.595}\right)^{0.76}\right]\right\}^{\frac{2}{3}},$$

where t is the thickness, c the chord, and A the aspect ratio.

For the case solved by Lugt (infinite aspect ratio, and a thickness ratio of 0.1) this equation predicts a tipspeed ratio of 0.63, or 40 % higher than the numerical solution. A further inference from this equation, were it to be valid outside the range of data from which it was derived, is that for a square plate ($t/c = 1.0$) the tipspeed ratio is zero irrespective of aspect ratio.

This paper examines experimental results which more closely approximate the conditions modelled numerically by Lugt than previous experimental studies. The rotating elements spanned the full width of the wind tunnel thereby approximating two-dimensional flow, and thus representing infinite aspect ratio plates. The work is extended beyond the limits of previous studies in that the full range of thickness ratios are examined, from thin plates ($t/c = 0.1$) to models of square cross-section ($t/c = 1.0$). The main properties of the plates that were examined were the ratio of the tipspeed (angular velocity × radius of swept area) to the free-stream velocity, and the lift and drag coefficients.

2. Apparatus

An open-circuit suction type wind tunnel with a maximum airspeed of 50 m/s was used for the laboratory tests. Working section dimensions were 400 mm × 100 mm and the model was placed centrally spanning the smaller dimension, as indicated schematically in figure 1. The flow was uniform over the central 90 % of the tunnel width and the gap between the sides of the plate and the tunnel wall was less than 1 mm. Free-stream turbulence level was less than 0.5 %. The models were attached to a yoke which formed part of a two-component force balance. A set of plates of 30, 40, 50 and 60 mm chord and thickness to chord ratios of 0.1, 0.3, 0.5, 0.7, 0.9 and 1.0 were tested. Tests with chords larger than 60 mm showed increasingly different trends, which were ascribed to the influence of tunnel blockage and were therefore discarded. The moment of inertia parameter was above the critical value for which it could be expected to have an influence (Iversen 1979).

As shown by Iversen (1979), the plate bearings are a critical factor in conducting experiments in this field. After experimentation with a variety of designs the following simple system was utilized. The plates were manufactured from a commercially available special purpose low-friction rigid polymer material (Vesconite), drilled both ends with a 1 mm diameter, 2 mm deep hole, on the rotational axis. Two highly polished tapered steel pins passing through the wind-tunnel walls are engaged loosely in these holes. A thin oil was applied to the bearing. The pins were rigidly attached to the yoke straddling the wind tunnel.

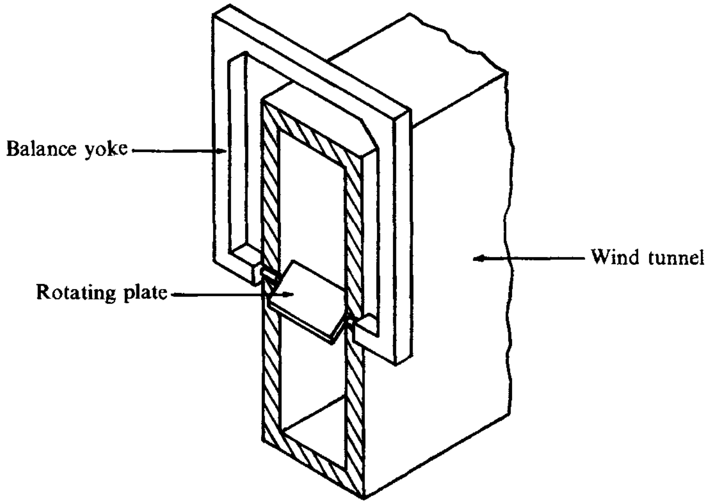

FIGURE 1. Schematic of plate mounting in the wind tunnel.

In all the tests the curves of tipspeed (V) against airspeed (U) extrapolated to the airspeed axis intersected at an airspeed of less than 2 m/s, or 4% of the maximum speed. This indicates the very low friction obtained, and can be compared to the curves given by Iversen (1979) for his own results and those of Smith (1971) and Glaser & Northup (1971). In these cases the velocity at intersection with the airspeed axis is between 10 and 25% of the maximum velocity. In analysing these latter results Iversen (1979) showed that the bearing friction can be largely ignored by taking for the value of V/U the slope $\mathrm{d}V/\mathrm{d}U$ of the curve. In the present tests this adjustment is applied, although the effect is very much less significant than for previous data.

It was found during the development of the bearing system, particularly for the narrower plates at the higher airspeeds where rotatonal velocities are high, that the bearings occasionally started to bind, resulting in a deviation from proportionality between tipspeed and airspeed. The tapered pins were removed and repolished when this occurred and the tests repeated. Insufficient data was obtained for the 30 mm $\times$ 3 mm plate as the flexure of the plate resulting from the aerodynamic loads pulled it free from the retaining pins, which after a few attempts damaged the pivot holes to the extent that the plate could not be used.

3. Results

Typical experimental results for thickness ratios of 0.1, 0.5 and 1.0 are given in figure 2. Iversen's correlation and Lugt's results are superimposed where applicable. The major point to note is that for a thickness ratio of 0.1 the agreement with Lugt's computation, is very good. On the other hand, the agreement with Iversen's correlation, for all thickness ratios, is poor. In particular this correlation does not allow for autorotation of square plates ($t/c = 1.0$) whereas in the experiments rotation was achieved as easily as for the thin plates. It is therefore apparent that Iversen's correlation does not correctly account for the effect of aspect ratio beyond a value of about 4.

Figure 3 shows the variation of tipspeed ratio obtained experimentally for the

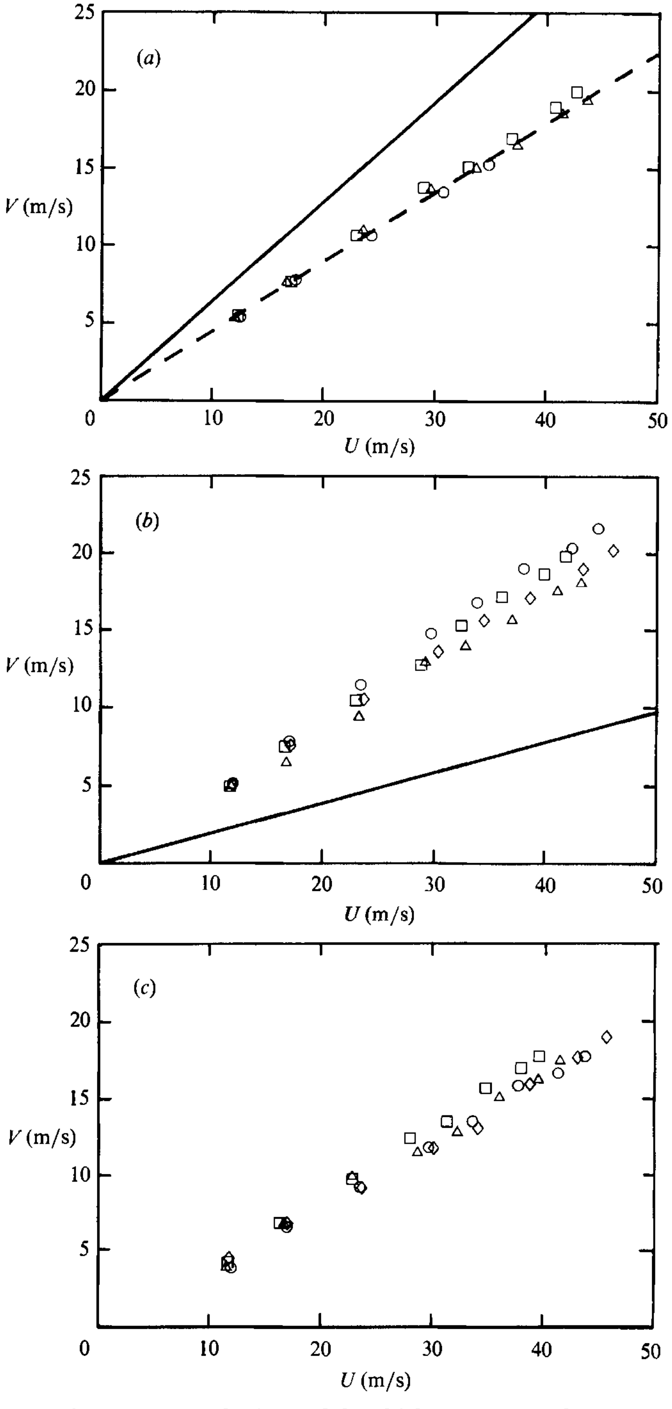

FIGURE 2. Tipspeed variation with airspeed for thickness ratios of (*a*) 0.1, (*b*) 0.5 and (*c*) 1.0, and various chords. ◇, 30 mm; ○, 40 mm; △, 50 mm; □, 60 mm; – – –, Lugt value; ——, Iversen's correlation.

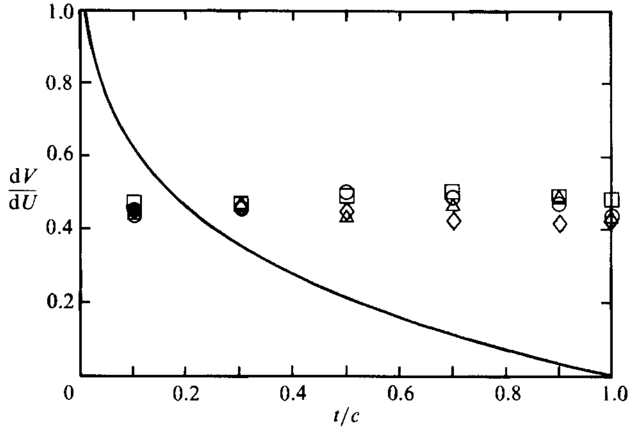

FIGURE 3. Tipspeed ratio variation with thickness ratio. ◇, 30 mm; ○, 40 mm; △, 50 mm; □, 60 mm; ●, Lugt value; ——, Iversen's correlation.

suite of plates tested. The values plotted are the slopes of the least squares line for the data. In all cases the correlation coefficient was higher than 0.995. A few tests were redone to check for repeatability. Although the correlation coefficient remained high for each test the range of tipspeed ratios obtained varied between 0.420 and 0.518 for all tipspeed ratios and plate chords, with no discernible pattern to the variation. It is concluded that slight differences in the mounting was responsible for this variation.

The surprising result is that the tipspeed ratio appears to be independent of thickness ratio, at approximately the Lugt value, over the whole range of thickness ratios, notwithstanding vortices been shed twice per revolution for very thin plates and presumably four times per revolution for the square section.

Within the accuracy of the measurement no effect of Reynolds number was apparent, which confirms the conclusion reached by both Iversen and Lugt.

Lift and drag coefficients are shown in figure 4 and are based on the diameter of the swept volume rather than the chord, as used in previous work. This change of definition has a small effect for thin plates.

The drag coefficient is also found to be independent of thickness ratio, with an indication of an increase in coefficient with an increase in chord. The average value of 1.35 is similar to that found by Iversen (1979) from his analysis of previous data. The lift coefficient shows a marked change between thickness ratios of 0.3 and 0.5. It is constant at a value of 1.3 for t/c of 0.1 and 0.3, and again constant at a value of 0.8 for thickness ratios 0.7, 0.9 and 1.0.

4. Discussion

Bustamante & Stone's experimental results, as taken from the graphs given by Iversen (1979), are compared with the present results for the 60 mm chord plate in figure 5, in order to indicate the effect of aspect ratio. Most of the quoted results are for very thin plates, thickness ratios generally being less than 0.15. Such thin plates could unfortunately not be used in the current tests because of the pivot design. It is unlikely that the tipspeed ratio for the two-dimensional tests would change significantly from the Lugt value for thickness ratios less than the 0.1 tested. This view is also expressed by Lugt. On this basis, for thin plates, it is noted that the

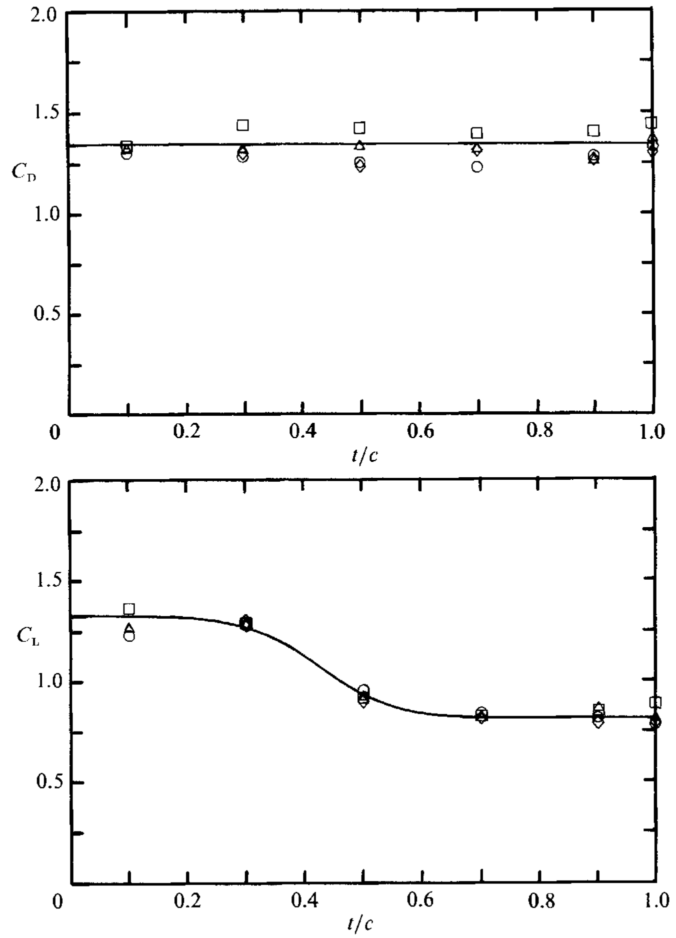

FIGURE 4. Force coefficient variation with thickness ratio. ◇, 30 mm; ○, 40 mm; △, 50 mm; □, 60 mm.

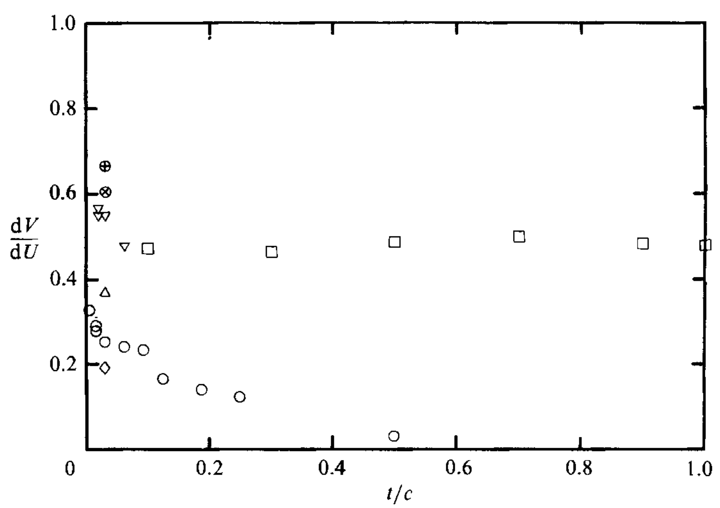

FIGURE 5. Tipspeed ratio variation with thickness ratio for various aspect ratios. ◇, 0.25; ○, 0.5; △, 1.0; ▽, 2.0; ⊗, 3.0; ⊕, 4.0; □, ∞.

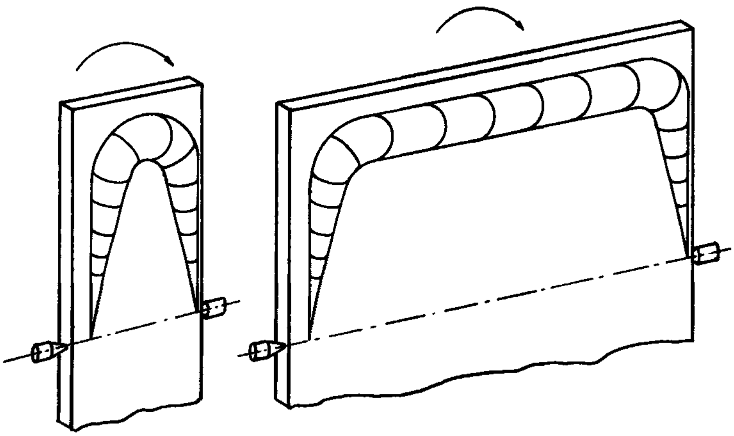

FIGURE 6. Schematic of vortices shed from the retreating edge.

tipspeed ratio increases from approximately 0.2 for an aspect ratio of 0.25, to 0.66 at an aspect ratio of 4, and, to satisfy the current data, would subsequently reduce to the Lugt value of 0.45 as the aspect ratio increases.

For very low aspect ratios the main vortices shed would be from the radial edges of the plate rather than from the spanwise edge as indicated schematically in figure 6(*a*). The moment arm of the resulting low-pressure regions about the rotational axis will thus be less than for higher-aspect-ratio plates (figure 6*b*) where the predominant vortex is that shed from the spanwise edge. The radial edge vortices would decrease in strength towards the rotational axis owing to the lower plate velocity there. They would also interact strongly because of their close proximity. The aerodynamic moment would thus be expected to be small. As the aspect ratio increases, contributions to the driving aerodynamic moment will arise from the main vortex from the retreating edge of the plate as well as that from the radial edges, resulting in a higher tipspeed than for the two-dimensional case. As the aspect ratio increases towards infinity so the influence of the radial edge vortices on the total moment will decrease. The noted increase of tipspeed with aspect ratio, followed by a subsequent reduction to the two-dimensional value thus appears reasonable on physical grounds.

Lugt has shown that the driving aerodynamic moment arises predominantly from the strong vortex shed from the retreating face of the plate. It thus occurs twice per revolution for a very thin plate. Although Lugts' calculations were for a thin elliptical section the presence of this vortex should not be significantly affected for a thin rectangular section. However, as the plate thickness increases relative to its width a stage would be reached where significant vortices would be shed from all four corners, and will thus occur four times per revolution. It is therefore even more surprising that the tipspeed ratio should remain unaffected. It is suggested that the sudden change in lift coefficient at a thickness ratio of about 0.4 arises owing to a change from two dominant vortices to four. Detailed flow-visualization studies or further numerical simulations would be needed to clarify the reason.

It should be noted that the models tested had a physical aspect ratio of between 1.7 and 3.3, and that the ends of the plates are imbedded in the wind-tunnel boundary layer. There is also a 1 mm gap between the end of the plate and the tunnel wall, as well as a small clearance (2 mm) between the support pivot and the wall through which a flow may be induced. It is clear that the flow at the tunnel wall in the vicinity of the plate will be complex, and that the plate will be driving part of the slower boundary-layer flow, rather than being driven as it as in the main flow. These end effects have not been quantified. A greater difference in results for the different widths of plates would have been expected if they were large.

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